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University Of Diyala – College Of Engineering

Digital Signal Processing

Course Instructor
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Lecture No. 3

Third Class
Department of Computer and Software Engineering

Lecture Outline

- Classification of Discrete Time Systems (DTS)
- Basic Operations on Signals
- Describing Digital Signals with Impluse Function

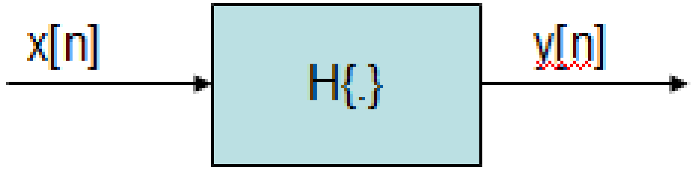
Discrete Time Systems (DTS)

- A discrete time system is a device or algorithm that operates on a discrete time signal x[n], called the input or excitation, according to some well defined rule, to produce another discrete time signal y[n] called the output or response of the system.
- We express the general relationship between x[n] and y[n] as

$$y[n] = H\{x[n]\}$$

where the symbol H denotes the transformation (also called an *operator*), or processing performed by the system on x[n] to produce y[n].

Discrete-time System



Static Versus Dynamic:

- Static System = memory less = the output doesn't depend on the past future values of the input.
- Dynamic system = having either finite or infinite memory.

Example 1:

$$y[n] = x^2[n]$$ Static or memory-less System

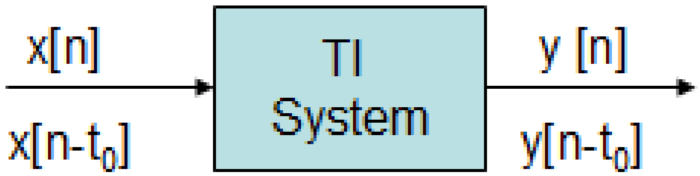
$$y[n] = \sum_{k=0}^{N} x[n-k]$$ Dynamic-finite

$$y[n] = \sum_{k=0}^{\infty} x[n-k]$$ Dynamic-infinite

Classification of Discrete Time Systems (DTS)

Time Invariant versus Time Variant Systems:

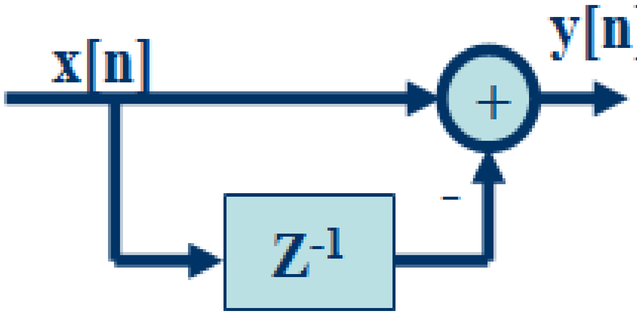
- A system is time invariant if
  - When the input is shifted in time, then its output is shifted by the same amount
  - This must hold for all possible shifts.
  - Stated in another way, *a system is called time invariant if its input-output characteristics do not change with time*. Otherwise the system is said to be time variant.
- If a shift in input $x[n]$ by t_0 causes a shift in output $y[n]$ by t_0 for all real-valued t_0, then system is time-invariant:



Classification of Discrete Time Systems (DTS)

Example 2: Determine if the system shown in the figure is time invariant or time variant.

Solution 2: $y[n] = x[n] - x[n-1]$



Now if the input is delayed by k units in time and applied to the system, the output is

$y[n,k] = n[n-k] - x[n-k-1]$ (1)

On the other hand, if we delay y[n] by k units in time, we obtain

$y[n-k] = x[n-k] - x[n-k-1]$ (2)

(1) and (2) show that the system is **time invariant**.

Examples

Example 3: Determine if the following systems are time invariant or time variant.

(a) $y[n] = nx[n]$ (b) $y[n] = x[n]\cos w_0 n$

Solution 3:

(a) The response to this system to $x[n-k]$ is

$$y[n,k] = nx[n-k] \quad (3)$$

Now if we delay $y[n]$ by k units in time, we obtain

$$y[n-k] = (n-k)x[n-k]$$
$$= nx[n-k] - kx[n-k] \quad (4)$$

which is different from (3). This means the system is time-variant.

(b) The response of this system to $x[n-k]$ is

$$y[n,k] = x[n-k]\cos w_0 n \quad (5)$$

If we delay the output $y[n]$ by k units in time, then

$$y[n-k] = x[n-k]\cos w_0[n-k]$$

which is different from that given in (5), hence the system is time variant.

Tutorials:

Q4: Determine whether the following systems are time invariant or time variant.

(a) $y[n] = y[n-1] + 2x[n] - 3x[n-1] + 2x[n-2]$

(b) $y[n] - (y[n-2])/n = 2x[n]$

Linearity

- A system is linear if it is both
  - *Homogeneous*: If we scale the input, then the output is scaled by the same amount:

$$f(a\,x(t)) = a\,f(x(t))$$

  - *Additive*: If we add two input signals, then the output will be the sum of their respective outputs

$$f(x_1(t) + x_2(t)) = f(x_1(t)) + f(x_2(t))$$

- A system that is both linear and time invariant is called Linear Time-Invariant (LTI) system.

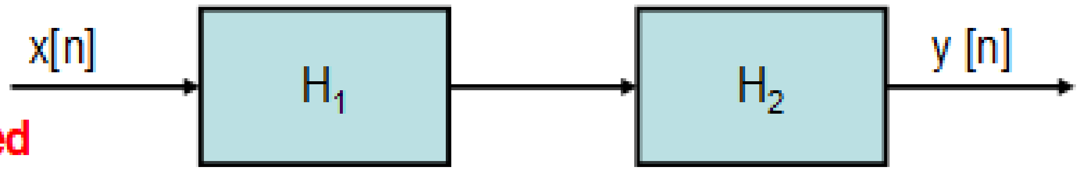
Linear versus Non-linear Systems:

A system H is linear if and only if

$$H[a_1x_1[n] + a_2x_2[n]] = a_1H[x_1[n]] + a_2H[x_2[n]]$$

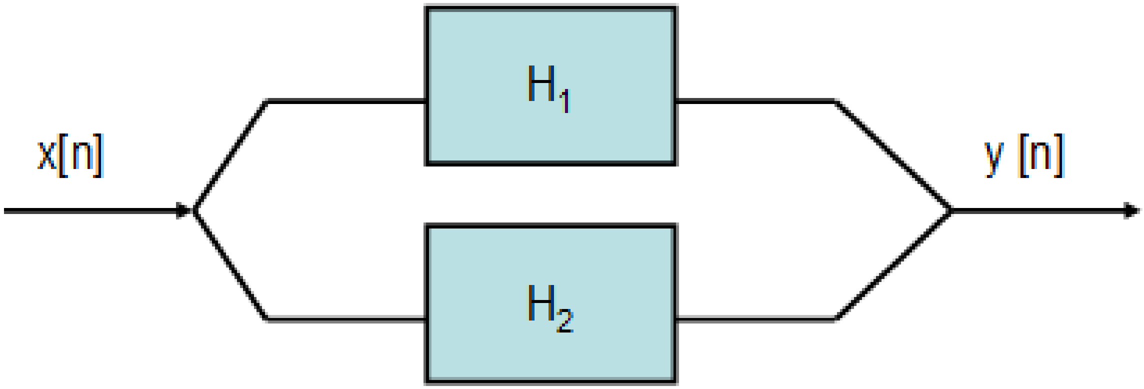
for any arbitrary input sequences $x_1[n]$ and $x_2[n]$, and any arbitrary constants a_1 and a_2.

Cascaded



$H=H_1 \cdot H_2=H_2 \cdot H_1$ for Linear systems only!

Parallel



$H=H_1+H_2$

for Linear systems only!

Examples

Example 4: Determine if the following systems are linear or nonlinear.

(a) $y[n] = n\,x[n]$ (b) $y[n] = A\,x[n] + B$

Solution 4:

(a) $y[n] = n\,x[n]$

For two input sequences $x_1[n]$ and $x_2[n]$, the corresponding outputs are

$$y_1[n] = nx_1[n] \quad \text{and} \quad y_2[n] = nx_2[n]$$

A linear combination of the two input sequences results in the output

$$H[a_1x_1[n] + a_2x_2[n]] = n[a_1x_1[n] + a_2x_2[n]] = na_1x_1[n] + na_2x_2[n] \qquad (1)$$

On the other hand, a linear combination of the two outputs results in the out

$$a_1y1[n] + a_2y_2[n] = a_1nx_1[n] + a_2nx_2[n] \qquad (2)$$

Since the right hand sides of (1) and (2) are identical, the system is linear.

Examples

(b) $y[n] = A\,x[n] + B$

Assuming that the system is excited by $x_1[n]$ and $x_2[n]$ separately, we obtain the corresponding outputs

$$y_1[n] = Ax_1[n] + B \quad \text{and } y_2 = Ax_2[n] + B$$

A linear combination of $x_1[n]$ and $x_2[n]$ produces the output

$$\begin{aligned} y_3[n] &= H[a_1x_1[n] + a_2x_2[n]] = A[a_1x_1[n] + a_2x_2[n]] + B \\ &= Aa_1x_1[n] + Aa_2x_2[n] + B \qquad (3) \end{aligned}$$

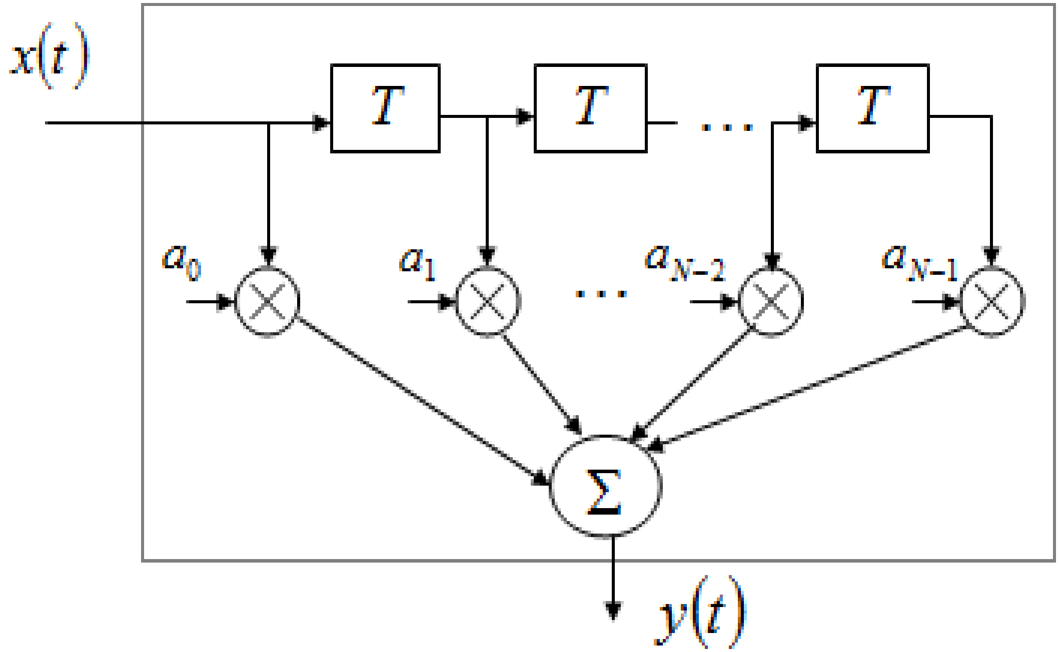
On the other hand, if the system were linear, its output to the linear combination of $x_1[n]$ and $x_2[n]$ would be a linear combination of $y_1[n]$ and $y_2[n]$, that is,

$$a_1y_1[n] + a_2y_2[n] = a_1Ax_1[n] + a_1B + a_2Ax_2[n] + a_2B \qquad (4)$$

Clearly, (3) and (4) are different and hence the system is nonlinear. Under what conditions would it be linear?

Examples

- Tapped delay line



Each T represents a delay of T time units

There are N-1 delays

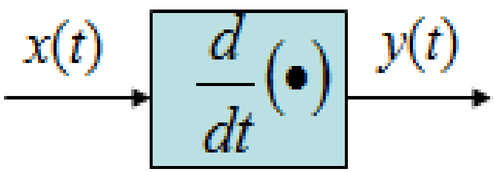
$$y(t) = a_0\, x(t) + a_1\, x(t - T) + \cdots + a_{N-1}\, x(t - (N-1)T) = \sum_{k=0}^{N-1} a_k\, x(t - kT)$$

Tutorial

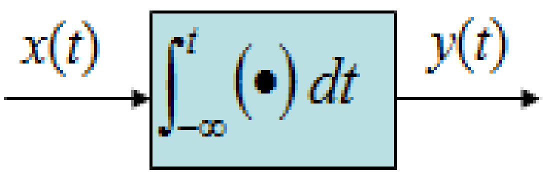
Q5: Determine whether following systems are linear or non linear.

(a) Squarer $y(t) = x^2(t)$

(b) Differentiation $y(t) = \frac{d}{dt}x(t)$



(c) Integration $y(t) = \int_{-\infty}^{t} x(u)\,du$



Causal versus Noncausal Systems

- A system is said to be causal if the output of the system at any time n [i.e. y[n]) depends only on present and past inputs [i-e x[n], x[n-1],…]but does not depend on future inputs [i.e. x[n+1], x[n+2]…]. If the system does not satisfy this definition, it is called noncausal.

Example: Determine if the systems described by the following input-output equations are causal or noncausal.

(a) $y[n] = x[n] - x[n-1]$ (b) $y[n] = ax[n]$

(c) $y[n] = x[n] + 3x[n+4]$ (d) $y[n] = x[n^2]$

(e) $y[n] = x[-n]$

Solution:

The systems (a), (b) are causal, all others are non-causal. $y[n]=x[-n]$ is non-causal because $y(-1)=x(1)$! Thus the o/p at n=-1 depends on the i/p at n=1, which is two units of time into the future.

Future value!

Classification of Discrete Time Systems (DTS)

Stable versus unstable Systems:

- A system is stable if any bounded input produces bounded output (BIBO).
- Otherwise, it is unstable!
- The condition that the i/p sequence x(n) & the o/p sequence y(n) are bounded is translated mathematically to mean that there exist some finite numbers.

Say M_x & M_y, such that

$|x(n)|<=M_x<inf \quad |y(n)|<=M_y<inf$ for all n.

If, for some reason bounded i/p sequence x(n), the o/p is unbounded (infinite), system is unstable.

Classification of Discrete Time Systems (DTS)

Invertibility:

- A system is said to be *invertible* if the input to the system may be uniquely determined from the output.
  - In order for a system to be invertible, it is necessary for distinct inputs to produce distinct outputs.
  - In other words, given any two inputs $x_1(n)$ and $x_2(n)$ with $x_1(n) \neq x_2(n)$, it must be true that $y_1(n) \neq y_2(n)$.
- This property is important in applications such as channel equalization and deconvolution is *invertibility*.

Example: The system defined by

$$y(n) = x(n)g(n)$$

Solution: is invertible if and only if $g(n) \neq 0$ for all n. In particular, given y(n) with g(n) nonzero for all n, x(n) may be recovered from y(n) as follows:

$$x(n) = \frac{y(n)}{g(n)}$$

Summary

- If several causes are acting on a linear system, then the total effect is the sum of the responses from each cause
- In time-invariant systems, system parameters do not change with time
- For memoryless systems, the system response at any instant *n* depends only on the present value of the input (value at *n*)
- If a system response at n depends on future input values (beyond n), then the system is noncausal.

Basic Operations on Signals

Operations performed on dependant variables:

- **Amplitude scaling**

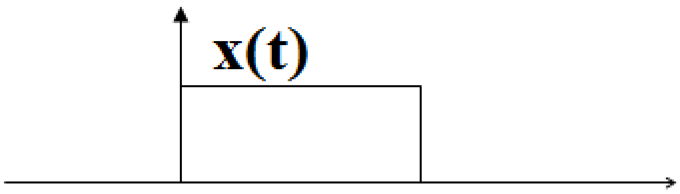
Let x(t) denote a continuous time signal. The signal y(t) resulting from amplitude scaling applied to x(t) is defined by

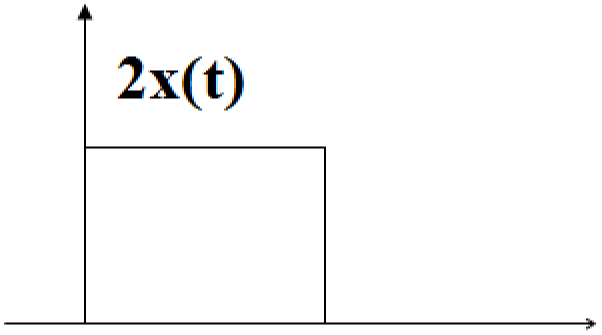
$$y(t) = c\,x(t)$$

where c is the scale factor.

In a similar manner to the above equation, for discrete time signals we can write

$$y[nT] = c\,x[nT]$$





Basic Operations on Signals

Addition

Let $x_1[n]$ and $x_2[n]$ denote a pair of discrete time signals. The signal $y[n]$ obtained by the addition of $x_1[n] + x_2[n]$ is defined as

$$y[n] = x_1[n] + x_2[n]$$

Example: Audio mixer

Multiplication

Let $x_1[n]$ and $x_2[n]$ denote a pair of discrete-time signals. The signal $y[n]$ resulting from the multiplication of the $x_1[n]$ and $x_2[n]$ is defined by

$$y[n] = x_1[n].x_2[n]$$

Example: AM Radio Signal

Basic Operations on Signals

Operations performed on independant variables:

- Time scaling

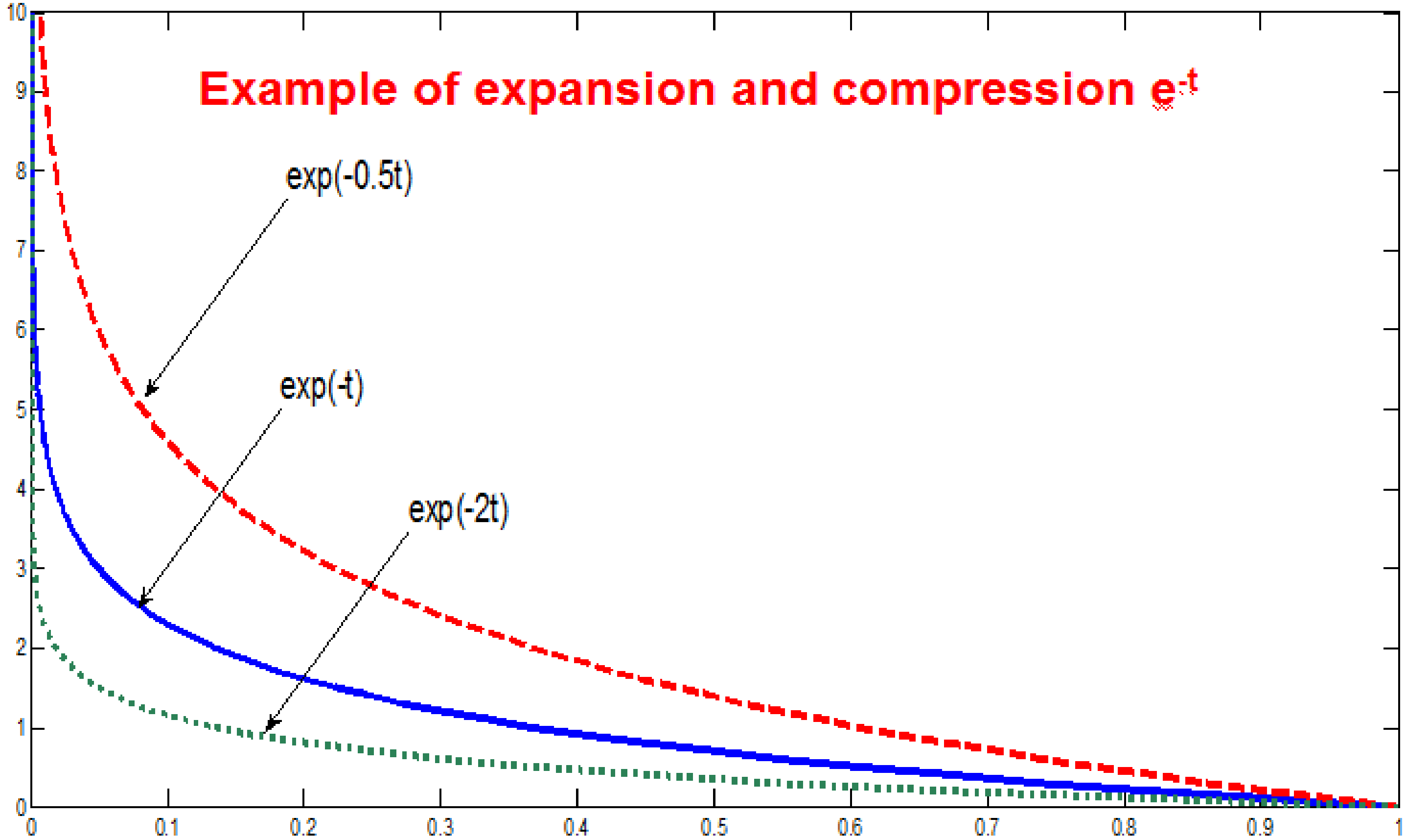
Let y(t) is a compressed version of x(t). The signal y(t) obtained by scaling the independent variable, time t, by a factor k is defined by

$$y(t) = x(kt)$$

- if $k > 1$, the signal y(t) is a compressed version of x(t).
- If, on the other hand, $0 < k < 1$, the signal y(t) is an expanded (stretched) version of x(t).

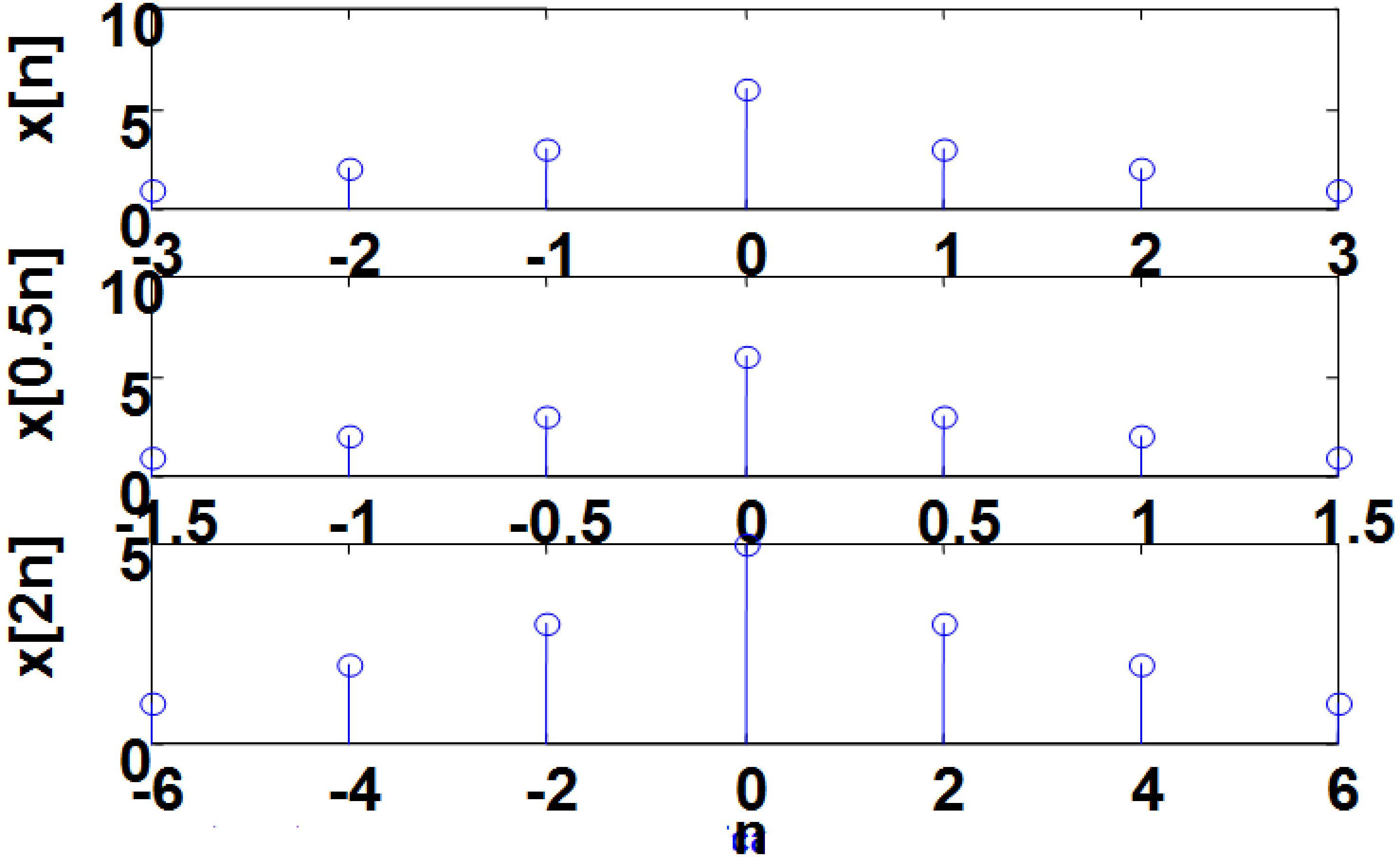
Basic Operations on Signals

Example:



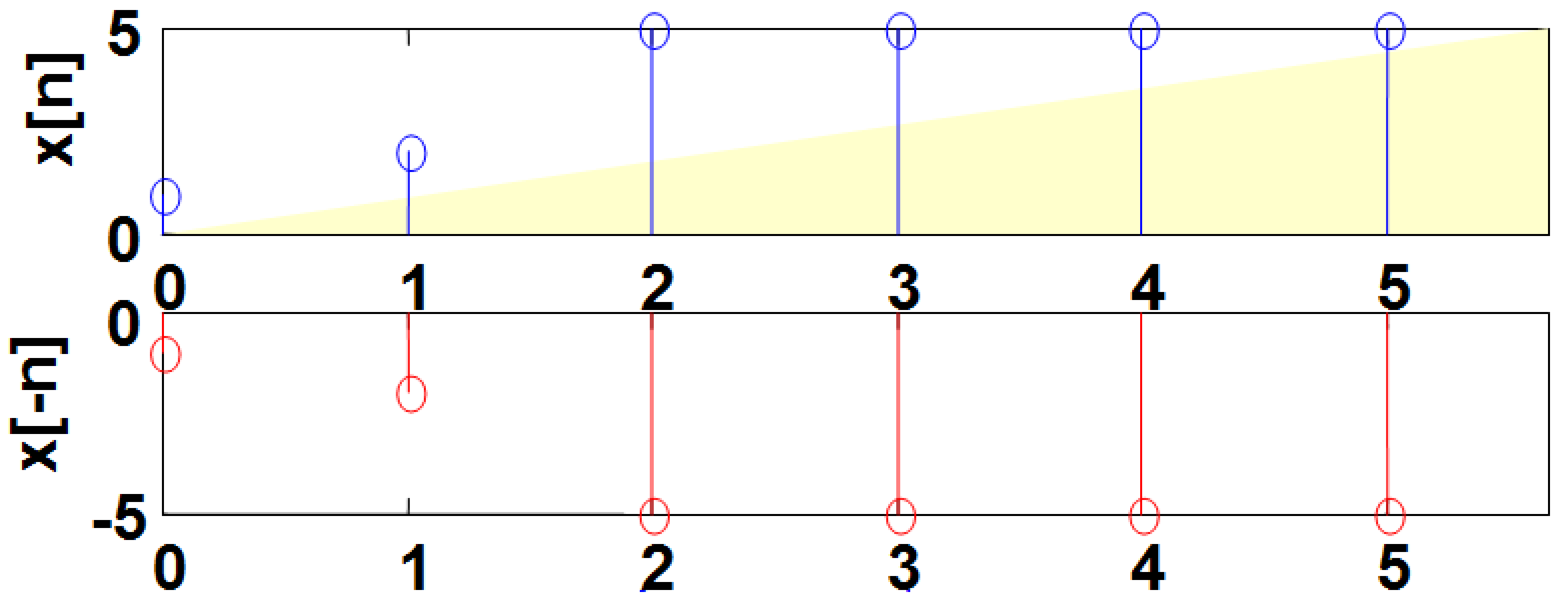
Basic Operations on Signals

Time scaling of discrete-time signals



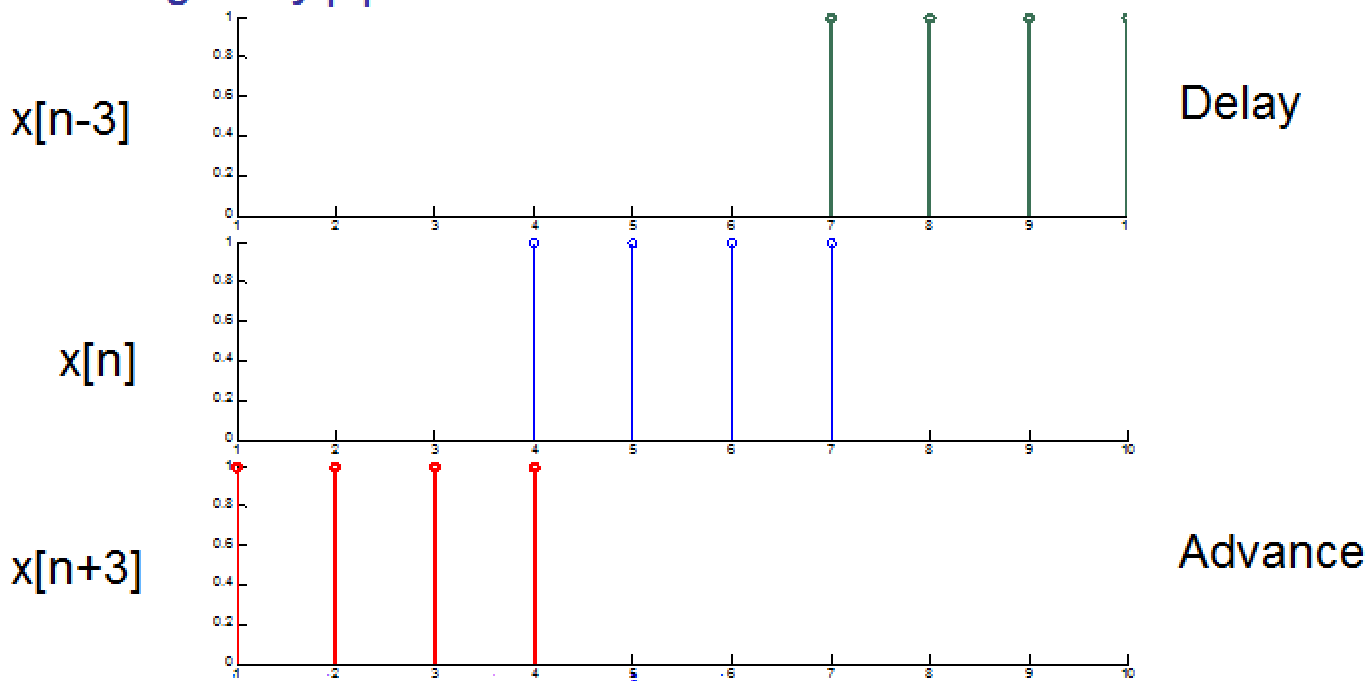
Basic Operations on Signals

- Time Reversal
  - This operation reflects the signal about t = 0 and thus reverses the signal on the time scale.



Time Shift

A signal may be shifted in time by replacing the independent variable n by n ± k, where k is an integer. If k is a positive integer, the time shift results in a delay of the signal by k units of time. If k is a negative integer, the time shift results in an advance of the signal by |k| units in time.

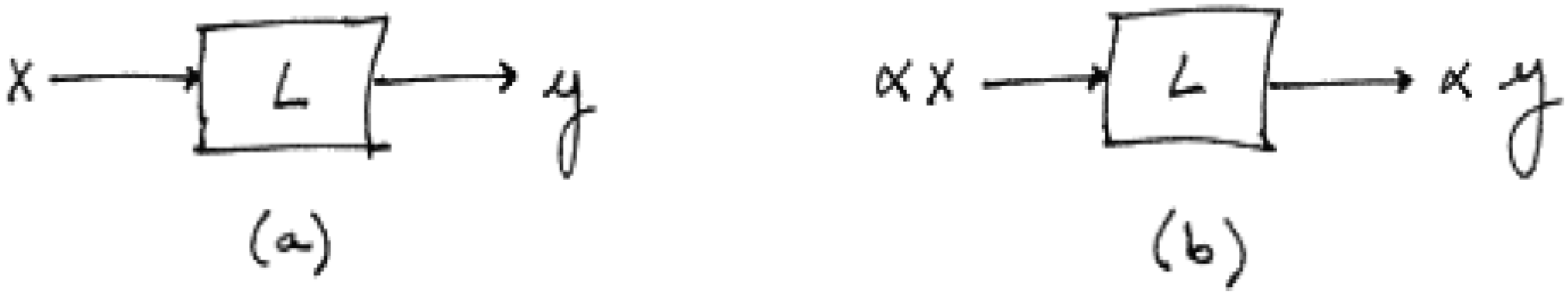


Linear Time-Invariant (LTI) Systems

- **Linear Systems**

If a system is linear, this means that when an input to a given system is scaled by a value, the output of the system is scaled by the same amount.

Linear Scaling



Figure

Linear Time-Invariant (LTI) Systems

- **Linear Systems**

If a system is linear, this means that when an input to a given system is scaled by a value, the output of the system is scaled by the same amount.

In part (a) of the figure above, an input x to the linear system L gives the output y If x is scaled by a value α and passed through this same system, as in part (b), the output will also be scaled by α.

A linear system also obeys the principle of superposition. This means that if two inputs are added together and passed through a linear system, the output will be the sum of the individual inputs' outputs.

That is, if (a) is true, then (b) is also true for a linear system. The scaling property mentioned above still holds in conjunction with the superposition principle. Therefore, if the inputs x and y are scaled by factors α and β, respectively, then the sum of these scaled inputs will give the sum of the individual scaled outputs:

Linear Time-Invariant (LTI) Systems

- **Linear Systems**

Superposition Principle

(a)

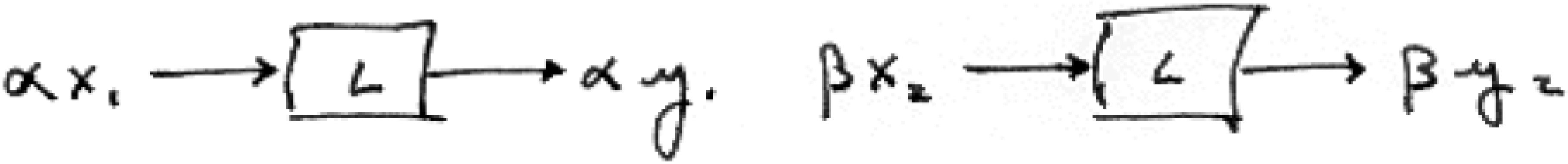
(b)

Figure : If (a) is true, then the principle of superposition says that (b) is true as well. This holds for linear systems.

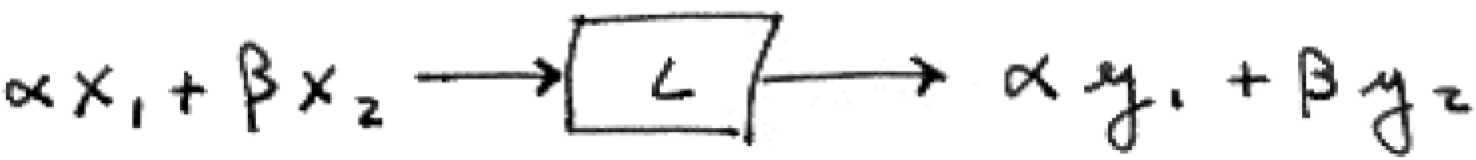
Linear Time-Invariant (LTI) Systems

- **Linear Systems**

Superposition Principle with Linear Scaling



(a)



(b)

Figure : Given (a) for a linear system, (b) holds as well.

Linear Time-Invariant (LTI) Systems

- **Time-Invariant Systems**

A time-invariant system has the property that a certain input will always give the same output, without regard to when the input was applied to the system.

In this figure, $x(t)$ and $x(t-t_0)$ are passed through the system TI. Because the system TI is time-invariant, the inputs $x(t)$ and $x(t-t_0)$ produce the same output. The only difference is that the output due to $x(t-t_0)$ is shifted by a time t_0.

Whether a system is time-invariant or time-varying can be seen in the differential equation (or difference equation) describing it. *Time-invariant systems are modeled with constant coefficient equations.* A constant coefficient differential (or difference) equation means that the parameters of the system are *not* changing over time and an input now will give the same result as the same input later.

Linear Time-Invariant (LTI) Systems

- **Time-Invariant Systems**

Time-Invariant Systems

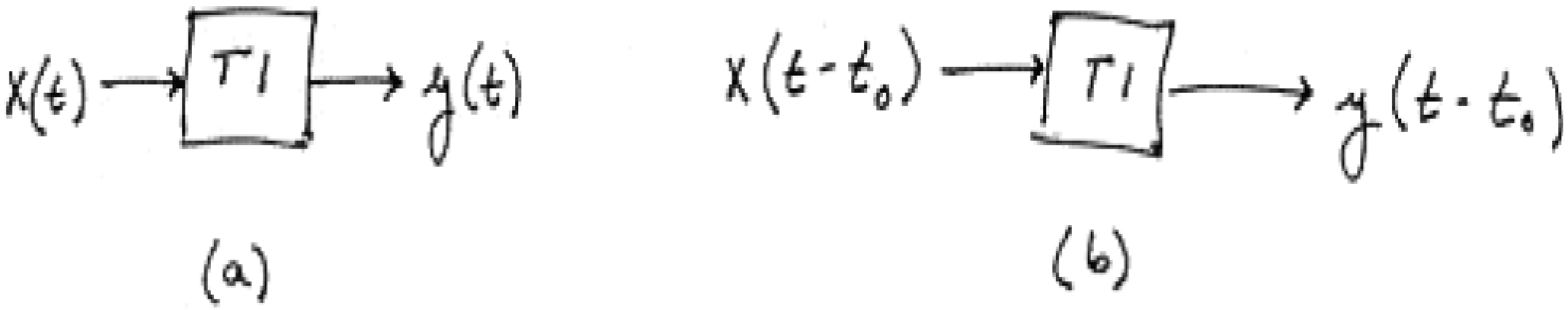


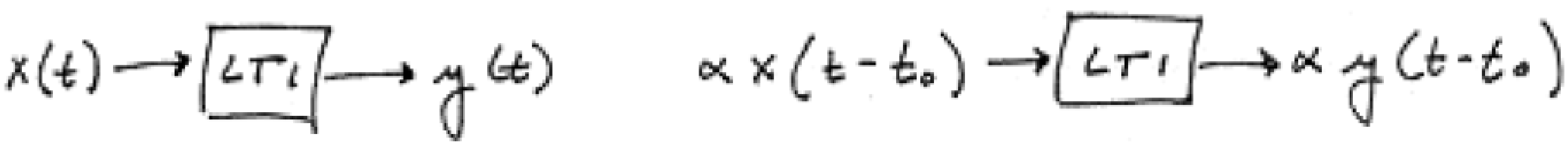
Figure : (a) shows an input at time t while (b) shows the same input t_0 seconds later. In a time-invariant system both outputs would be identical except that the one in (b) would be delayed by t_0.

Linear Time-Invariant (LTI) Systems

Certain systems are both linear and time-invariant, and are thus referred to as LTI systems.

As LTI systems are a subset of linear systems, they obey the principle of superposition. In the figure below, we see the effect of applying time-invariance to the superposition definition in the linear systems section above.

Linear Time-Invariant Systems



(a) (b)

Figure : This is a combination of the two cases above. Since the input to (b) is a scaled, time-shifted version of the input in (a), so is the output.

Linear Time-Invariant (LTI) Systems

Superposition in Linear Time-Invariant Systems

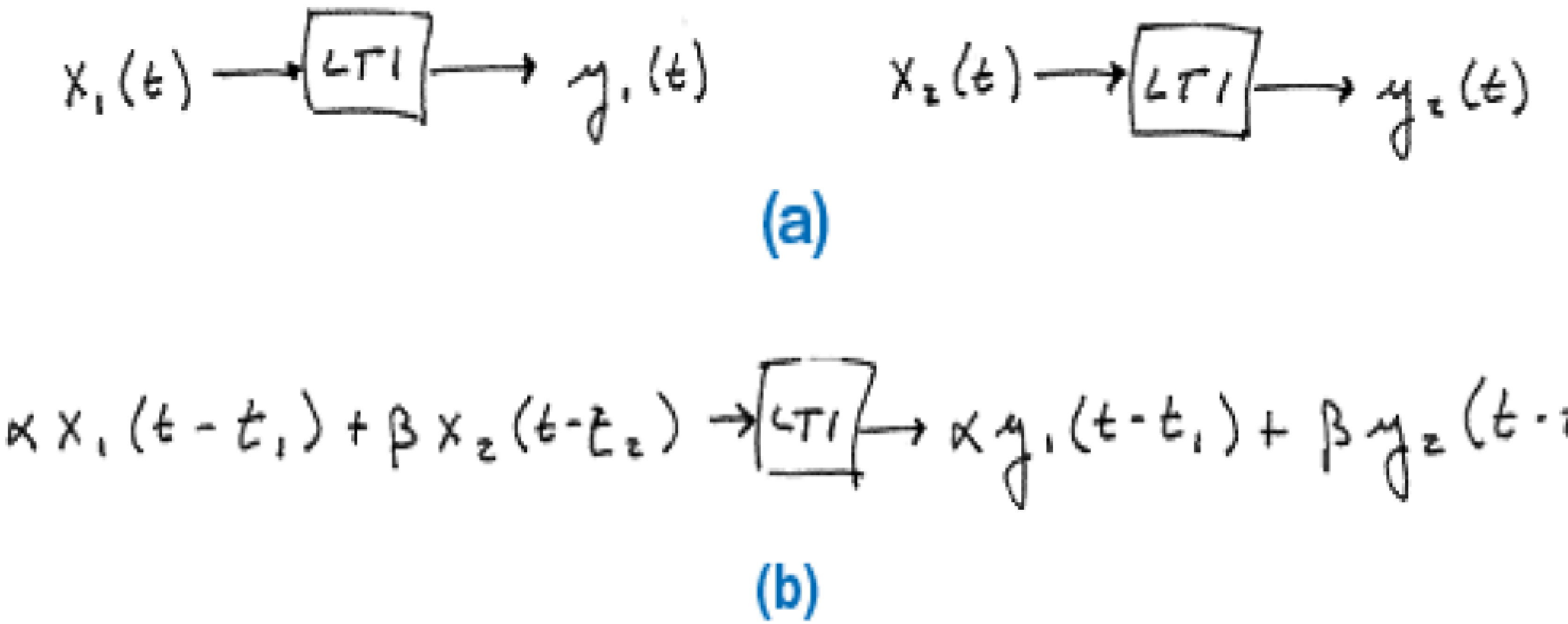


Figure : The principle of superposition applied to LTI systems

Linear Time-Invariant (LTI) Systems

"LTI Systems in Series"

If two or more LTI systems are in series with each other, their order can be interchanged without affecting the overall output of the system. Systems in series are also called cascaded systems.

"LTI Systems in Parallel"

If two or more LTI systems are in parallel with one another, an equivalent system is one that is defined as the sum of these individual systems.

Linear Time-Invariant (LTI) Systems

Cascaded LTI Systems

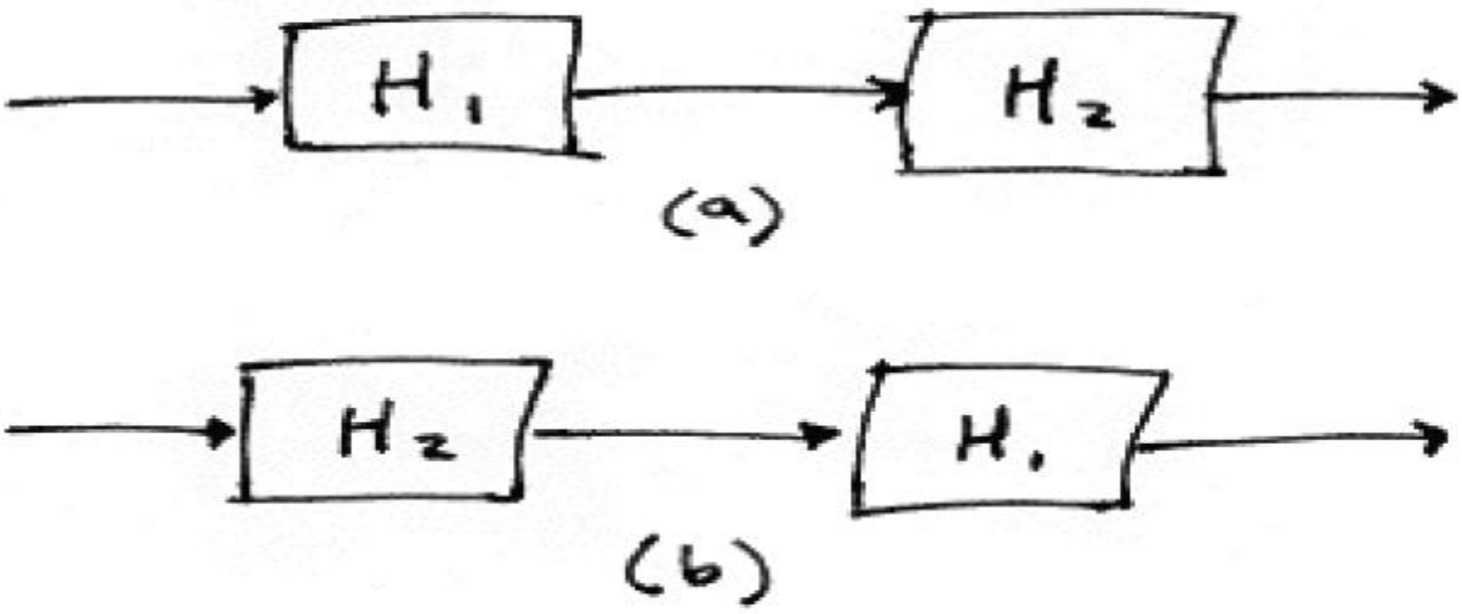


Figure : The order of cascaded LTI systems can be interchanged without changing the overall effect.

Linear Time-Invariant (LTI) Systems

Parallel LTI Systems

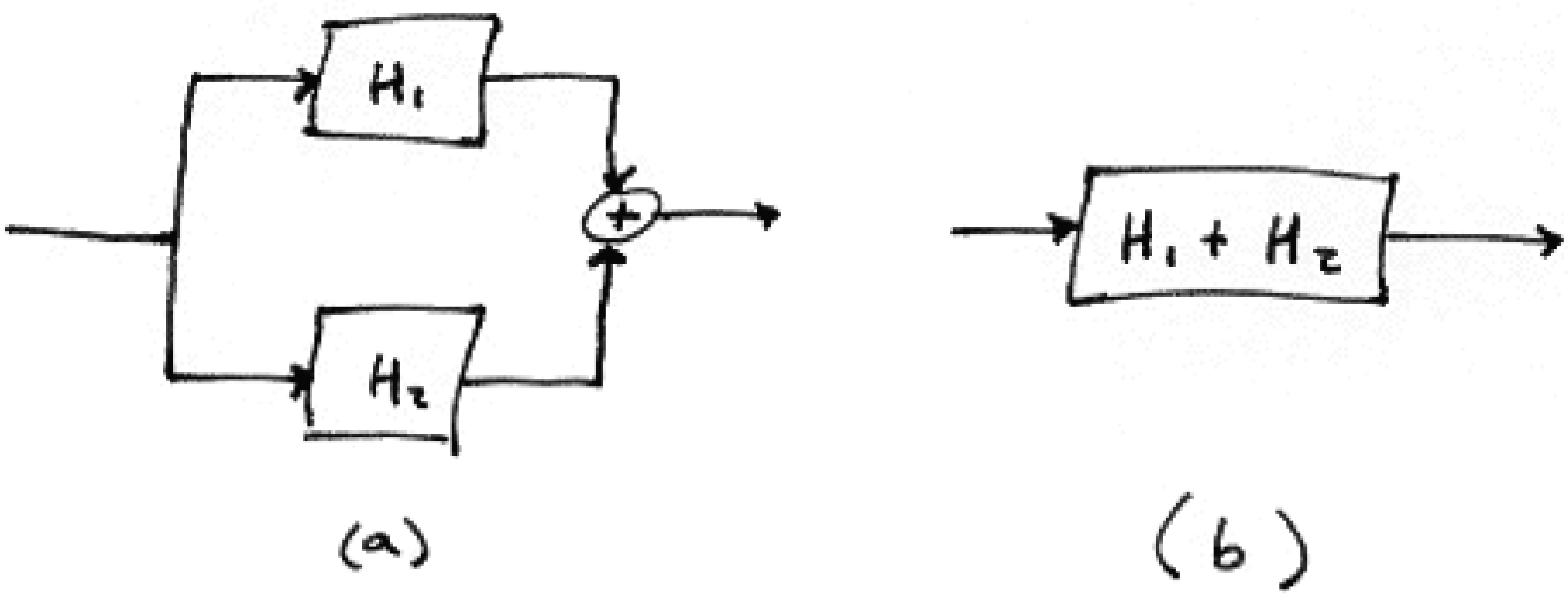


Figure : Parallel systems can be condensed into the sum of systems.